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Matrix Newton Interpolation and Progressive 3D Imaging: PC-Based Computation

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Abstract—For polynomials $P(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$ in a real scalar x, but with coefficients A_j that are rectangular matrices, a generalization of Newton's divided difference interpolatory scheme is developed. Instances of $P(x)$ at nodes x_i may be interpreted as slices of a digital 3D object. *Mathematica* code for this machinery is given and its effectiveness illustrated for progressively-transmitted renderings. Analysis, with supporting *Mathematica* code, is extended to a piecewise matrix polynomial situation, to produce practicable software for a PC-based computational system. Two experiments about 3D progressive imaging, employing a 6 Mbyte data base consisting of 93 CT slices of a human head, are discussed along with PC-based performance evaluation. How a 3D object is decomposed into 2D subsets in preparation for progressive transmission, as well as their selected ordering for transmission, are seen to affect quality of the emerging reconstructions. Extension to 4D objects is also discussed briefly, to provide introduction to, for example, application of matrix polynomial machinery within the field of functional magnetic resonance imaging. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords—Progressive transmission of images, Matrix Newton interpolation, Matrix polynomial reconstruction, PC-based progressive rendering.

1. INTRODUCTION

Modern medical imaging began with Roentgen's discovery of X-rays, just over 100 years ago, and much of the rapid development in the science has occurred in the last 25 years. As the technology has matured, the quality and quantity of data produced in a typical examination have increased dramatically. For example, a computerized tomography (CT) exam of the early 1970s usually consisted of one or two slices and perhaps 100 kilobytes of data. By the early 1980s, exams

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