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Matrix Cubic Splines for Progressive 3D Imaging*

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Abstract. Mathematical theory of matrix cubic splines is introduced, then adapted for progressive rendering of images. 2D subsets of a 3D digital object are transmitted progressively under some ordering scheme, and subsequent reconstructions using the matrix cubic spline algorithm provide an evolving 3D rendering. The process can be an effective tool for browsing three dimensional objects, and effectiveness is illustrated with a test data set consisting of 93 CT slices of a human head. The procedure has been implemented on a single processor PC system, to provide a platform for full 3D experimentation; performance is discussed. A web address for the complete, documented *Mathematica* code is given.

Keywords: progressive 3D rendering, progressive transmission of images, matrix cubic splines, matrix polynomial reconstructions

1. Introduction

Modern medical imaging began with Roentgen's discovery of X-rays, just over 100 years ago, and much of the rapid development in the science has occurred in the last 25 years. As the technology has matured, the quality and quantity of data produced in a typical examination have increased dramatically. For example, a Computerized Tomography (CT) exam of the early 1970's usually consisted of one or two slices and perhaps 100 kilobytes of data. By the early 1980's, exams contained $10-20$ slices and $1-2$ megabytes (MB) of data, and by the end of the decade, 50 or more slices with 25 MB of data per exam were common.

Current spiral-CT scanners can image hundreds of slices per exam and produce 100 MB or more of data, while Magnetic Resonance or 3D Ultrasound imaging is capable of capturing 500 MB of data, in just a few minutes. Other imaging fields have also seen burgeoning amounts of data, as user requirements become more sophisticated: in general interactive video, for example, even picture browsing currently imposes major limits on computational and storage resources, while developments for full interactivity remain in the realm of virtual reality technologies R & D.

In medical as well as other application areas of imaging science, differing approaches for handling ever-increasing amounts of 3D data are under investigation. One broad class of techniques impose some compression scheme on the full digital object and, for network transmission, the compressed image is not rendered until all transmission is finished; see for example [8] and [10] for a wavelet approach, or [9] and [17] for further mathematical structures. A second set of techniques, and

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